

34th Austrian Mathematical Olympiad
Federal Competition for Advanced Students
Part 1, May 28, 2003

1. Determine all triples (p, q, r) of primes such that $p^q + p^r$ is a square number.
2. Determine the smallest and largest possible value of $f(x, y) = y - 2x$ for all non-negative real numbers x, y with $x \neq y$ and $\frac{x^2+y^2}{x+y} \leq 4$.
3. Let t be a positive real number. Determine the number of positive real solutions (a, b, c, d) of the following system of equations.

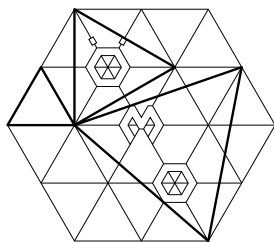
$$a(1 - b^2) = t$$

$$b(1 - c^2) = t$$

$$c(1 - d^2) = t$$

$$d(1 - a^2) = t$$

4. In a parallelogram $ABCD$, let E be the midpoint of the side AB and F the midpoint of BC . Let P be the intersection point of the lines EC and FD .
Show that the segments AP, BP, CP and DP divide the parallelogram into four triangles with areas in $1 : 2 : 3 : 4$ ratio.

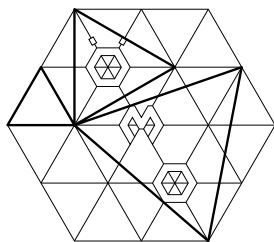


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Part 2, Day 1, June 28, 2003

1. Let the polynomial $P(n) = n^3 - n^2 - 5n + 2$ be given.
Determine all integers n such that $P(n)^2$ is the square of a prime.
2. Let a, b, c be real numbers different from zero such that there exist $\alpha, \beta, \gamma \in \{-1, 1\}$ with $\alpha a + \beta b + \gamma c = 0$
What is the smallest possible value of $\left(\frac{a^3+b^3+c^3}{abc}\right)^2$.
3. Around each grid point (x, y) with non-negative integers coordinates a square with the grid point as its center and length of its sides equal to $\frac{0,9}{2^{x5^y}}$ is drawn in arbitrary orientation.
Determine the area of this figure consisting of an infinit number of squares.



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Part 2, Day 2, June 29, 2003

4. Show that for each base $g > 2$ there exists exactly one three-digit number $(abc)_g$, that is represented as $(cba)_h$ with the digits in reverse order in a base h that differs from g by 1.
5. We are given a sufficient amount of bricks: Rectangles of the size 2×1 and squares of the size 1×1 .

Let $n > 3$ be a natural number.

How many possibilities exist to fill a $3 \times n$ rectangle with these bricks in such a way that all 2×1 rectangles have their longer sides parallel to the side of length 3 and do not touch each other?

6. Let ABC be an acute-angled triangle. The circle k with diameter AB intersects the lines AC and BC in the points P and Q . Let R be the intersection point of the tangents in A and Q and let S be the intersection point of the tangents in B and P .

Show that C lies on the line RS .