32nd Austrian Mathematical Olympiad

Regional Competition for Advanced Students

April 24, 2001

1. Let n be an integer and $S(n)$ be the sum of the 2001 powers of n with exponents 0 through 2000. That is, $S(n) = \sum_{k=0}^{2000} n^k$.
   Determine the final digit (i.e., the ones-digit) in the decimal expansion of $S(n)$.

2. Determine all real solutions of the equation

$$(x+1)^{2001} + (x+1)^{2000}(x-2) + (x+1)^{1999}(x-2)^2 + \cdots \\ + (x+1)^2(x-2)^{1999} + (x+1)(x-2)^{2000} + (x-2)^{2001} = 0$$

3. In a convex pentagon $ABCDE$ the areas of the triangles ABC, ABD, ACD and ADE are all equal to the same value F. What is the area of the triangle BCE?

4. Let $A_0 = \{1, 2\}$ and for $n > 0$ let A_n be the set of all numbers that are either elements of A_{n-1} or can be represented as the sum of two distinct elements of A_{n-1}.
   Further let $a_n = |A_n|$ be the number of elements of A_n.
   Determine a_n as a function of n.