34th Austrian Mathematical Olympiad

Regional Competition for Advanced Students

April 29, 2003

1. Determine the smallest possible value of $\frac{a+1}{a(a+2)}+\frac{b+1}{b(b+2)}+\frac{c+1}{c(c+2)}$ for positive real numbers a, b, c with $a+b+c \le 3$.

2. Determine all primes p such that $5^p + 4p^4$ is a square number.

3. We are given two parallel lines g and h and a point P lying outside the stripe between g and h. Three mutually distinct lines g_1, g_2 and g_3 are drawn through P, intersecting g in the points A_1, A_2, A_3 and h in the points B_1, B_2, B_3.
   The points $C_{12} = (A_1B_2) \cap (A_2B_1)$, $C_{13} = (A_1B_3) \cap (A_3B_1)$, $C_{23} = (A_2B_3) \cap (A_3B_2)$ are the intersection points of the corresponding connections.
   Show that
   (a) there exists exactly one line n containing the points C_{12}, C_{13}, and C_{23} and
   (b) n is parallel to g and h.

4. For each real number b determine all real numbers x such that $x - b = \sum_{k=0}^{\infty} x^k$ holds.