



43rd Austrian Mathematical Olympiad
Regional Competition for Advanced Students
April 19, 2012

1. Prove that the inequality

$$a + a^3 - a^4 - a^6 < 1$$

holds for all real numbers a .

2. Determine all integer solutions (x, y) of the equation

$$(x - 1)x(x + 1) + (y - 1)y(y + 1) = 24 - 9xy.$$

3. In an arithmetic sequence, the difference of consecutive terms is constant.

We consider sequences of integers in which the difference of consecutive terms equals the sum of the differences of all preceding consecutive terms.

Which of these sequences with $a_0 = 2012$ and $1 \leq d = a_1 - a_0 \leq 43$ contain square numbers?

4. In the triangle ABC let H_a , H_b and H_c denote the base points of the altitudes on the sides BC , CA , AB .

Determine for which triangles two of the lengths H_aH_b , H_bH_c and H_aH_c are equal?