



32nd Austrian Mathematical Olympiad

Beginner's Competition

June 7, 2001

1. Prove that for all odd positive integers n the number $n^n - n$ is divisible by 24.
2. We consider the quadratic equation $x^2 - 2mx - 1 = 0$, where m is an arbitrary real number.
For which values of m does the equation have two real solutions, such that the sum of their cubes equals eight times their sum.
3. Determine all real numbers x such that the inequality

$$(x - 1)^2(x - 4)^2 < (x - 2)^2$$

holds.

4. Let ABC be a triangle whose angles $\alpha = \angle CAB$ and $\beta = \angle CBA$ are greater than 45° .
Above the side AB we construct a right-angled isosceles triangle ABR with AB as hypotenuse, such that R lies *inside* the triangle ABC .
Analogously we erect above the sides BC and AC right-angled isosceles triangles CBP and ACQ , but with their (right-angled) vertices P and Q *outside* of the triangle ABC .
Show that $CQRP$ is a parallelogram.