36th Austrian Mathematical Olympiad

Beginner's Competition

June 16, 2005

1. Show that there are no positive integers a and b such that $4a(a+1) = b(b+3)$.

2. Determine the number of integer pairs (x, y) such that

$$(|x| - 2)^2 + (|y| - 2)^2 < 5.$$

3. Determine all triples (x, y, z) of real numbers that satisfy all of the following three equations.

$$\lfloor x \rfloor + \{y\} = z$$
$$\lfloor y \rfloor + \{z\} = x$$
$$\lfloor z \rfloor + \{x\} = y$$

   (For a real number u, $\lfloor u \rfloor$ is the largest integer smaller than or equal to u and $\{u\} = u - \lfloor u \rfloor$.)

4. We are given the triangle ABC with an area of 2000. Let P, Q, R be the midpoints of the sides BC, AC, AB. Let U, V, W be the midpoints of the sides QR, RP, PQ. The lengths of the line segments AU, BV, CW are x, y, z.

   Show that there exists a triangle with side lengths x, y and z and calculate its area.