39^{th} Austrian Mathematical Olympiad

Beginner's Competition

June 17^{th}, 2008

1. Determine all positive integers n such that
$$\frac{2^n}{n^2}$$
is an integer.

St. Wagner, Stellenbosch, South Africa

2. Determine all real numbers x satisfying
$$x\lfloor x\lfloor x\rfloor\rfloor = \sqrt{2}.$$
Here, $\lfloor x\rfloor$ denotes the largest integer less or equal to x.

Th. Eisenkölbl, Lyon, France

3. Prove the inequality
$$\frac{a+b}{a^2-ab+b^2} \leq \frac{4}{|a+b|}.$$
for all real numbers a and b with $a+b \neq 0$. When does equality hold?

K. Czakler, Vienna

4. Let ABC be an acute-angled triangle ABC with the property that the bisector of $\angle BAC$, the altitude through B and the perpendicular bisector of AB intersect in one point. Determine the angle $\alpha = \angle BAC$.

W. Janous, Innsbruck