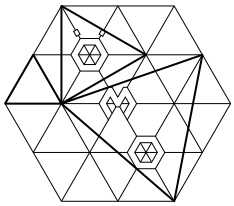

41st Austrian Mathematical Olympiad

Beginner's Competition

June 15th, 2010

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1. Show that 2010 cannot be written as the difference of two squares.

   B. Schmidt, Graz

2. We consider a group of trees in a nature reserve, all of which have a positive integral age. The average age is 41 years. After destruction of a tree with an age of 2010 years by lightening, the average age of the remaining trees is 40 years.

   Determine the original number of trees in the group. What is the maximal number of trees of an age of 2010 years in the original group?

   W. Janous, Innsbruck

3. Let x and y be positive real numbers with $x + y = 1$.

   Prove that

   $$\frac{(3x-1)^2}{x} + \frac{(3y-1)^2}{y} \geq 1.$$

   When does equality hold?

   K. Czakler, Vienna

4. Let ABC be a right-angled triangle with the right angle at C such that that the side BC is longer than the side AC. The perpendicular bisector of AB intersects the line BC in D and the line AC in E. We assume that DE and the side AB have the same length.

   Determine the angles of the triangle ABC.

   R. Henner, Vienna