



43th Austrian Mathematical Olympiad

Beginner's Competition

June 11th, 2012

1. Let a , b , c and d be four integers such that

$$7a + 8b = 14c + 28d.$$

Prove that $a \cdot b$ is a multiple of 14.

W. Janous, Innsbruck

2. A postman has n parcels of weights $1, 2, 3, 4, \dots, n$. He wants to divide the parcels into three groups of equal weight. Is this possible for

- (a) $n = 2011$,
(b) $n = 2012$?

G. Woeginger, Eindhoven, The Netherlands

3. Let a and b be positive real numbers with $a \leq 2b \leq 4a$.

Prove that

$$4ab \leq 2(a^2 + b^2) \leq 5ab.$$

G. Anegg, Innsbruck

4. The two equilateral triangles ABC and ADB (with $C \neq D$) share the common side AB . The midpoints of AC and BC are denoted by E and F , respectively.

Show that DE and DF divide AB into three parts of equal length.

G. Kirchner, Innsbruck