

# 44<sup>th</sup> Austrian Mathematical Olympiad

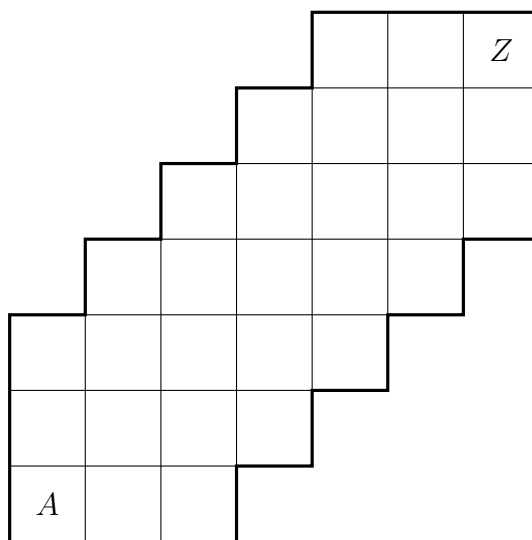
Beginners' Competition

June 13<sup>th</sup>, 2013

1. Find all integers  $n > 1$  such that the sum of  $n$  and its second-largest divisor is 2013.

*R. Henner, Vienna*

2. Find the number of paths from square  $A$  to square  $Z$  in the figure below where a path consists of steps from a square to its upper or right neighbouring square.



*W. Janous, WRG Ursulinen, Innsbruck*

3. Let  $a$  and  $b$  be real numbers with  $0 \leq a, b \leq 1$ .

Prove that

$$\frac{a}{b+1} + \frac{b}{a+1} \leq 1$$

and find the cases of equality.

*K. Czakler, Vienna*

4. Let  $ABC$  be an acute triangle and  $D$  be a point on the altitude through  $C$ .

Prove that the mid-points of the line segments  $AD$ ,  $BD$ ,  $BC$  and  $AC$  form a rectangle.

*G. Anegg, Innsbruck*