

45th Austrian Mathematical Olympiad

Beginners' Competition

June 12th, 2014

1. Determine all solutions of the Diophantine equation

$$a^2 = b \cdot (b + 7)$$

in integers $a \geq 0$ and $b \geq 0$.

W. Janous, Innsbruck

2. All empty white triangles in Figure 1 are to be filled with integers such that for each gray triangle the three numbers in the white neighboring triangles sum to a multiple of 5. The lower left and the lower right white triangle are already filled with the numbers 12 and 3, respectively.

Find all integers that can occur in the uppermost white triangle.

G. Woeginger, Eindhoven, The Netherlands

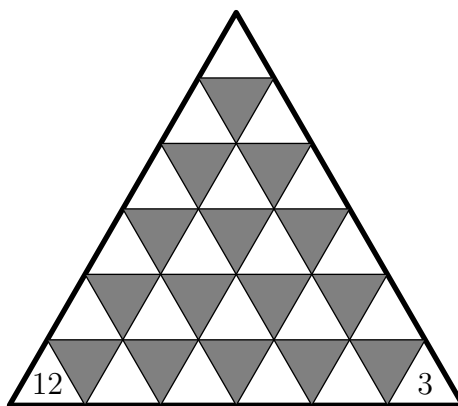


Figure 1: Problem 2

3. Let a, b, c and d be real numbers with $a < b < c < d$.

Sort the numbers $x = a \cdot b + c \cdot d$, $y = b \cdot c + a \cdot d$ and $z = c \cdot a + b \cdot d$ in ascending order and prove the correctness of your result.

R. Henner, Vienna

4. Consider a triangle ABC . The midpoints of the sides BC , CA , and AB are denoted by D , E , and F , respectively.

Assume that the median AD is perpendicular to the median BE and that their lengths are given by $\overline{AD} = 18$ and $\overline{BE} = 13.5$.

Compute the length of the third median CF .

K. Czakler, Vienna