47^{th} Austrian Mathematical Olympiad

National Competition (Final Round, part 1)

April 30, 2016

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1. Determine the largest constant C such that

$$(x_1 + x_2 + \cdots + x_6)^2 \geq C \cdot (x_1(x_2 + x_3) + x_2(x_3 + x_4) + \cdots + x_6(x_1 + x_2))$$

   holds for all real numbers $x_1, x_2, \ldots, x_6$.

   For this C, determine all $x_1, x_2, \ldots, x_6$ such that equality holds.

   (Walther Janous)

2. We are given an acute triangle ABC with $AB > AC$ and orthocenter H. The point E lies symmetric to C with respect to the altitude AH. Let F be the intersection of the lines EH and AC. Prove that the circumcenter of the triangle AEF lies on the line AB.

   (Karl Czakler)

3. Consider 2016 points arranged on a circle. We are allowed to jump ahead by 2 or 3 points in clockwise direction.

   What is the minimum number of jumps required to visit all points and return to the starting point?

   (Gerd Baron)

4. Determine all composite positive integers n with the following property: If $1 = d_1 < d_2 < \ldots < d_k = n$ are all the positive divisors of n, then

$$(d_2 - d_1) : (d_3 - d_2) : \cdots : (d_k - d_{k-1}) = 1 : 2 : \cdots : (k-1).$$

   (Walther Janous)

Working time: $4\frac{1}{2}$ hours.
Each problem is worth 8 points.