



54th Austrian Mathematical Olympiad

Regional Competition

30th March 2023

1. Let a, b and c be real numbers with $0 \leq a, b, c \leq 2$. Prove that

$$(a - b)(b - c)(a - c) \leq 2.$$

When does equality hold?

(Karl Czakler)

2. Let $ABCD$ be a rhombus with $\angle BAD < 90^\circ$. The circle passing through D with center A intersects the line CD a second time in point E . Let S be the intersection of the lines BE and AC .

Prove that the points A, S, D and E lie on a circle.

(Karl Czakler)

3. Determine all natural numbers $n \geq 2$ with the property that there are two permutations (a_1, a_2, \dots, a_n) and (b_1, b_2, \dots, b_n) of the numbers $1, 2, \dots, n$ such that $(a_1 + b_1, a_2 + b_2, \dots, a_n + b_n)$ are consecutive natural numbers.

(Walther Janous)

4. Determine all pairs (x, y) of positive integers such that for $d = \gcd(x, y)$ the equation

$$xyd = x + y + d^2$$

holds.

(Walther Janous)

Working time: 4 hours.

Each problem is worth 8 points.