51st Austrian Mathematical Olympiad

Regional Competition

2nd April 2020

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1. Determine all positive integers a for which the equation

$$\left(1+\frac{1}{x}\right)\cdot\left(1+\frac{1}{x+1}\right)\cdots\left(1+\frac{1}{x+a}\right)=a-x$$

has at least one integer solution x.

For each such integer a, determine the corresponding solutions.

(Richard Henner)

2. The set M consists of all 7-digit positive integers which contain each of the digits 1, 3, 4, 6, 7, 8 and 9 (in base 10) exactly once.

   a) Determine the smallest positive difference d between any two numbers in M.

   b) How many pairs (x, y) with x and y in M exist for which $x - y = d$ holds?

(Gerhard Kirchner)

3. Let ABC be a triangle with $AB < AC$ and incenter I. The perpendicular bisector of the side BC intersects the angle bisector of $\angle BAC$ at the point S, and the angle bisector of $\angle CBA$ at the point T, respectively.

Show that the points C, I, S and T lie on a common circle.

(Karl Czakler)

4. Determine all quadruples (p, q, r, n) which satisfy the equation

$$p^2 = q^2 + r^n$$

where p, q, r are prime numbers and n is a positive integer.

(Walther Janous)

Working time: 4 hours.
Each problem is worth 8 points.