51^{st} Austrian Mathematical Olympiad

Junior Regional Competition

6th June 2020

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1. Determine all pairs (a, b) of real numbers satisfying the inequality

$$\frac{(1+a)^2}{1+b} \leq 1 + \frac{a^2}{b}$$

where $b \neq -1$ and $b \neq 0$. For which pairs (a, b) does equality hold?

(Walther Janous)

2. How many five-digit numbers exist with the property that the product of the digits of each number equals 900?

(Karl Czakler)

3. Let $ABCD$ be an isosceles trapezoid with $AB \parallel CD$ and $AB > CD$. Let E be the foot of the perpendicular from D onto the line AB and let M be the mid-point of the diagonal BD.

Prove that the lines EM and AC are parallel.

(Karl Czakler)

4. Determine all positive integers a for which the equation

$$7an - 3n! = 2020$$

has a solution n in positive integers.

(*Note*: For every positive integer n: $\quad n! = 1 \cdot 2 \cdot \cdots \cdot n$.)

(Richard Henner)

Working time: 4 hours.
Each problem is worth 8 points.