



53rd Austrian Mathematical Olympiad

Junior Regional Competition

14th June 2022

1. Show that for all real numbers x and y with $x > -1$, $y > -1$ and $x + y = 1$ the inequality

$$\frac{x}{y+1} + \frac{y}{x+1} \geq \frac{2}{3}$$

holds.

When does equality hold?

(Walther Janous)

2. Consider a 13×2 rectangular board and an arbitrarily large number of dominoes in sizes 2×1 and 3×1 .

We want to cover the board with dominoes without gaps or overlaps or parts of a domino outside the board. Additionally, all dominoes have to have the same orientation, i. e. all their longer sides have to be parallel.

How many configurations are possible?

(Walther Janous)

3. Consider the semicircle with center M and diameter AB . Let P be a point on the semicircle different from A and B and let Q be the midpoint of the arc AP . Let S be the intersection of the line BP with the parallel of PQ through M .

Prove that $PM = PS$.

(Karl Czakler)

4. Find all primes p , q and r with $p + q^2 = r^4$.

(Karl Czakler)

Working time: 4 hours.

Each problem is worth 8 points.